

Date - 04/02/2025

MJC-7
OPTICS

B.SC (Sem. IV)

Principle of Superposition of two waves

Let us consider two simple harmonic collinear vibrations given by

$$y_1 = a_1 \sin \omega t \quad \text{--- (1)}$$

$$\text{and } y_2 = a_2 \sin(\omega t + \delta) \quad \text{--- (2)}$$

Where y_1, y_2 are displacements of a particle at time t under the action of two disturbances having a_1 and a_2 respective amplitudes

ωt and $(\omega t + \delta)$ are phase angles

δ = phase difference between two vibrations.

ω = frequency

According to the principle of superposition y_1 and y_2 acts independently and their resultant is the algebraic sum of the two i.e.

$$\begin{aligned} y &= y_1 + y_2 = a_1 \sin \omega t + a_2 \sin(\omega t + \delta) \\ &= a_1 \sin \omega t + a_2 \sin \omega t \cos \delta + a_2 \cos \omega t \sin \delta \\ &= (a_1 + a_2 \cos \delta) \sin \omega t + a_2 \sin \delta \cos \omega t \quad \text{--- (3)} \end{aligned}$$

Let $a_1 + a_2 \cos \delta = A \cos \phi$ and $a_2 \sin \delta = A \sin \phi$

From eqn. (3),

$$y = A \sin(\omega t + \phi) \quad \text{--- (4)}$$

Where A and ϕ are the constants depending on a_1, a_2 and δ

Eqn (4) shows that the resultant displacement has same frequency

squaring and adding component relations of (4), we get.

$$A^2 = a_1^2 + a_2^2 + 2a_1 a_2 \cos \delta$$

$$A = \sqrt{a_1^2 + a_2^2 + 2a_1 a_2 \cos \delta}$$

Again dividing the components,

$$\frac{A \sin \phi}{A \cos \phi} = \frac{a_1 \sin \delta}{a_1 + a_2 \cos \delta}$$

$$\therefore \tan \phi = \frac{a_1 \sin \delta}{a_1 + a_2 \cos \delta} \quad \text{--- (6)}$$

Equation (5) and (6) gives the values of the amplitude and the phase of the resultant vibrations.

Since intensity is proportional to the square of the amplitude, therefore, $I = A^2 = a_1^2 + a_2^2 + 2a_1 a_2 \cos \delta$

For intensity to be max; $\cos \delta = +1$ then

$$\delta = 0, 2\pi, 4\pi, \dots = 2n\pi \text{ with } n = 0, 1, 2, 3, \dots$$

$$\boxed{I_{\max}} = a_1^2 + a_2^2 + 2a_1 a_2 = (a_1 + a_2)^2 \quad \text{--- (7)}$$

$$\text{Max}^m \text{ Amplitude } A_{\max} = a_1 + a_2$$

For minimum intensity, $\delta = -1$, then

$$\delta = \pi, 3\pi, 5\pi, \dots = (2n+1)\pi$$

where $n = 0, 1, 2, 3, \dots$

$$\text{And } \boxed{I_{\min}} = a_1^2 + a_2^2 - 2a_1 a_2 = (a_1 - a_2)^2$$

In terms of path difference

For Max^m Intensity $\Delta \phi = 2n\lambda$ and

Min Intensity $\Delta \phi = (2n+1)\lambda/2$

special case

$$\text{When } a_1 = a_2, \quad \boxed{I_{\max} = 4a^2 \text{ and } A_{\max} = 2a}$$

$$I_{\min} = 0 \text{ and } A_{\min} = 0$$

$$\frac{I_{\max}}{I_{\min}} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2} \text{ and } \frac{A_{\max}}{A_{\min}} = \frac{a_1 + a_2}{a_1 - a_2} = \frac{2a}{0} = \infty$$

$$= \frac{4a^2}{0} = \infty \text{ and}$$

$$\boxed{\frac{I_{\max}}{I_{\min}} = \infty \text{ and } \frac{A_{\max}}{A_{\min}} = \infty}$$

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